

## A Nonparametric Side-Sensitive Synthetic Control Chart using Signed-Rank Statistic

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### Abstract

We propose the nonparametric two-sided synthetic and side-sensitive synthetic control charts using signed-rank statistic. Performance of the proposed nonparametric control charts is measured using the measure average time to signal. The Markov chain approach is employed to compute average time to signal of the proposed side-sensitive synthetic control chart. The proposed nonparametric side-sensitive synthetic control chart perform significantly better than the two-sided synthetic control chart as well as the two-sided synthetic control chart gives better performance in terms of average time to signal than the nonparametric control chart based on the signed-rank statistic. The usefulness of the proposed control chart is explored by using a numerical example.

**Keywords:** Nonparametric, Control chart, Signed-rank statistic, Side-sensitive, Average run length.

### Introduction

In the parametric control chart, it is assumed that the process output follows a specific probability distribution. If the process observations do not follow a specific probability distribution, then the performance of such control charts can be degraded. Such considerations provide reasons for development and applications of distribution-free control charts. The application of nonparametric or distribution-free control charts does not require knowledge of the specific probability distribution. If the run length distribution of the chart does not depend on the underlying process distribution when there is no shift in the process parameter under study, the chart is said to be distribution-free or nonparametric chart. Bakir and Reynolds (1979) developed a chart based on within-group ranking. Hackl and Ledolter (1991) provided a control chart based on ranks. Amin *et al.* (1995) suggested nonparametric quality control charts for monitoring process median and process dispersion. (Bakir, 2004) proposed a control chart based on the signed-rank statistic. Chakraborti and Eryilmaz (2007) developed a nonparametric Shewhart-type signed-rank control chart based on runs. Chakraborti *et al.* (2001) reported an overview and some results of nonparametric control charts. Human *et al.* (2010) proposed nonparametric control charts based on runs of sign statistic. Mukharji *et al.* (2013) proposed distribution-free exceedance cusum control charts for location. Liu *et al.* (2014) proposed a dual nonparametric cusum control chart based on ranks.

Khilare and Shirke (2015) studied the steady-state behavior of nonparametric control charts using sign statistic. Riaz and Abbasi (2016) proposed a double EWMA control chart for process monitoring. Abid *et al.* (2016) reported the use of ranked set sampling in nonparametric EWMA control charts based on the sign test statistic. Abid *et al.* (2016) proposed a nonparametric EWMA control chart based on the Wilcoxon signed-rank statistic for monitoring location. Pawar *et al.* (2018) studied the steady-state behavior of nonparametric synthetic control chart based on signed-rank statistic. Pawar *et al.* (2018) proposed the nonparametric moving average control charts using sign and signed-rank statistics. Chakraborti and Graham (2019) have taken review of both univariate and multivariate nonparametric control charts. Hence, considering the above facts this study, we propose the nonparametric two-sided synthetic and side-sensitive synthetic control charts using signed-rank statistic.

### Materials and methods

This study presents two-sided synthetic and side-sensitive synthetic control charts for monitoring the median of a continuous characteristic of the underlying process. The main purpose of the synthetic control chart is to improve the performance of the existing nonparametric control chart based on the signed-rank statistic for a wide class of process distributions.

Rest of the study is organized as follows:

In Section 2 control chart based on signed-rank statistic is described. Section 3 gives conforming run length control chart. In Section 4 a two-sided synthetic control chart is described. Section 5 gives side-sensitive synthetic control chart and its average run length using the Markov chains approach. Comparison study of the proposed control charts discussed in Section 6. Sections 7 and 8 respectively give numerical example and conclusions.

### Control chart based on the signed-rank statistic

Let  $(X_{t1}, X_{t2}, \dots, X_{tn})$  be a random sample of size  $n > 1$  observed from a continuous process with median  $\theta$ . It is assumed that the process distribution is continuous symmetric and that the in-control process median is specified to be equal to  $\theta_0$ . We further assume that when  $\theta \neq \theta_0$  the process is out-of-control. Bakir (2004) developed a nonparametric control chart based on the signed-rank statistic. For the  $t^{\text{th}}$  subgroup sample  $(x_{t1}, x_{t2}, \dots, x_{tn})$ , the signed-rank statistic is defined as

$$\psi_t = \sum_{j=1}^n \text{sign}(x_{tj} - \theta_0) R_{tj}^+, \quad t=1, 2, \dots \quad (1)$$

where  $\text{sign}(u) = -1, 0, 1$  if  $u < 0, = 0, > 0$  and

$$R_{tj}^+ = 1 + \sum_{i=1}^n I(|x_{ti} - \theta_0| < |x_{tj} - \theta_0|), \quad j=1, 2, \dots, n,$$

with  $I(a < b) = 1$ , if  $a < b$  and 0 otherwise.

We can rewrite (1) as

$$\psi_t = 2w_t^+ - \frac{n(n+1)}{2}, \quad (2)$$

Where  $w_t^+$  is the well-known Wilcoxon Signed-rank Statistic. Hereafter  $\psi_t$  given in (2) is used as a charting statistic instead of using (1). When  $n=1$ ,  $\psi_t$  takes values  $+1$  or  $-1$ . Therefore, it is not feasible to construct the chart in this case. Let UCL be the upper control limit corresponding to a positive-sided control chart. The chart gives an out-of-control signal at the first sampling instance  $t$  for which  $\psi_t \geq UCL$ . In the following section, we briefly describe CRL control chart.

### The conforming run length control chart

Bourke (1991) proposed the conforming run length (CRL) chart. In 100% inspection, the CRL is the number of inspected units between two consecutive non-conforming units (including the ending non-conforming unit). The CRL chart uses the CRL as the charting statistic. The idea behind the CRL chart is that the conforming run length will change when the fraction non-conforming 'p' in the process changes.

The charting statistic (CRL) follows a geometric distribution with parameter  $p$ . The average number of inspected units in a CRL sample is given by

$$\mu_{CRL} = \frac{1}{p} \quad (3)$$

and the cumulative distribution function (c. d. f) of CRL is,

$$F_p(CRL) = 1 - (1-p)^{CRL}; \quad CRL = 1, 2, \dots \quad (4)$$

If our interest only in the detection of an increase in  $p$ , the lower control limit (L) is sufficient for the CRL chart. If  $\alpha_{CRL}$  is the type I error of the CRL chart and  $p_0$  is the in-control fraction non-conforming, L can be derived from the following equation.

$$\alpha_{CRL} = F_{p_0}(L) = 1 - (1-p_0)^L,$$

$$\text{which gives} \quad L = \frac{\ln(1-\alpha_{CRL})}{\ln(1-p_0)}. \quad (5)$$

If L is not an integer, it is rounded to the nearest integer. If the sample CRL is less than or equal to L, process signals an out-of-control status. Let  $ARL_{CRL}$  be the average number of CRL samples required detecting a change in  $p$ , then it is given by

$$ARL_{CRL} = \frac{1}{p(1-(1-p)^L)} \quad (6)$$

In the following subsection, we describe two-sided synthetic control chart obtained by combining CRL chart described here and nonparametric control chart due to Bakir (2004).

### Two-sided synthetic control chart using signed-rank statistic

Wu and Spedding (2000) developed the synthetic control chart for detecting shifts in the process mean, which combines the Shewhart  $\bar{X}$  chart and CRL chart. The idea of combining a classical control chart with CRL chart has shown improvement in the ARL performance of many classical charts. In nonparametric setup (Khilare and Shirke, 2010; 2012) developed nonparametric synthetic control charts for process location and variability based on the sign statistic. Pawar and Shirke (2010) proposed the positive-sided synthetic control chart for process location based on the signed-rank statistic.



Table 1. In control ARL values for the two-sided synthetic chart for various values of UCL and L when n=8.

UCL	L									
	1	2	3	4	5	6	7	8	9	10
1	1.12	1.06	1.06	1.06	1.06	1.06	1.06	1.06	1.06	1.06
3	1.40	1.21	1.19	1.19	1.19	1.19	1.19	1.19	1.19	1.19
5	1.82	1.44	1.37	1.35	1.35	1.35	1.35	1.35	1.35	1.35
7	2.44	1.79	1.64	1.59	1.57	1.56	1.56	1.56	1.56	1.56
9	3.34	2.30	2.02	1.91	1.86	1.84	1.84	1.83	1.83	1.83
11	4.71	3.06	2.57	2.37	2.27	2.22	2.20	2.19	2.18	2.17
13	6.82	4.22	3.42	3.06	2.87	2.77	2.70	2.67	2.65	2.63
15	10.24	6.07	4.74	4.12	3.78	3.58	3.45	3.37	3.31	3.28
17	16.00	9.14	6.92	5.85	5.24	4.87	4.62	4.45	4.32	4.24
19	26.21	14.53	10.69	8.82	7.73	7.03	6.55	6.21	5.96	5.78
21	45.39	24.51	17.61	14.21	12.20	10.89	9.98	9.31	8.81	8.43
23	83.59	44.21	31.15	24.66	20.80	18.25	16.46	15.13	14.12	13.33
25	163.84	85.25	59.11	46.08	38.30	33.14	29.48	26.76	24.66	22.99
27	334.37	171.88	117.78	90.77	74.60	63.85	56.19	50.47	46.04	42.51
29	655.36	334.21	227.21	173.76	141.72	120.39	105.18	93.79	84.95	77.90
31	1820.44	921.02	621.26	471.43	381.56	321.68	278.93	246.89	221.98	202.08
33	4096.00	2064.13	1386.89	1048.31	845.20	709.82	613.15	540.66	484.30	439.23
35	16384.00	8224.13	5504.22	4144.31	3328.40	2784.49	2396.00	2104.66	1878.08	1696.83

The main purpose of the synthetic chart is to improve the performance of the existing nonparametric control chart based on the signed-rank statistic for a wide class of process distributions. The proposed nonparametric two-sided synthetic control chart is a combination of the nonparametric signed-rank control chart based on  $\psi_t$  and the CRL chart.

**Operation**

The operation of the nonparametric synthetic control chart is as follows:

1. Decide on the upper control limit UCL and lower control limit LCL of the  $\psi_t$  chart and the lower control limit L of the CRL chart.
2. At each inspection point 't' take a random sample of n observations and calculate  $\psi_t$ .
3. If  $LCL < \psi_t < UCL$ , the sample is conforming sample and control flow goes back to step (2) Otherwise, the sample is called a nonconforming sample and control flow goes to the next step.
4. Check the number of samples between the current and the last nonconforming sample including the current sample and this number is taken as the CRL value in the synthetic chart.
5. If this CRL is larger than the L, then the process is under control and the charting procedure is continued. Otherwise, the process is declared to be out-of-control and control flow goes to the next step.
6. Take the necessary action to find and remove the assignable cause(s).

**Design**

The synthetic chart has three parameters namely L, UCL and LCL. For a given in-control ARL and subgroup sample size n, the control parameters L, UCL and LCL are obtained as follows:

Let  $p(\delta) = \Pr(\psi_t \leq LCL \text{ OR } \psi_t \geq UCL / \theta = \theta_0 + \delta)$ ,  $\delta \geq 0$ . It is clear that,  $p(\delta)$  is the probability that the sample is nonconforming when the shift of  $\delta$  units occurs in the process median. When there is no shift,  $\delta$  is equal to zero. We note that the in-control ARL of the synthetic chart is given by  $ARLs(0)$ , where

$$ARLs(0) = \frac{1}{p(0)(1 - (1 - p(0))^L)}, \tag{7}$$

We note that in equation (6), 'p' is the probability that a unit is nonconforming, while  $p(\delta)$  defined above is the probability that the sample is nonconforming. Thus  $p(\delta)$  plays the role of p in equation (6).

The two-sided synthetic control chart signals, when  $\psi_t \leq -c$  or  $\psi_t \geq c$  and  $CRL \leq L$  where c and -c are respectively the upper and lower control limits of the two-sided  $\psi_t$  chart while L is the lower limit of the CRL chart.

We compute the  $ARLs(0)$  values using equation (7) for  $UCL=1,2,\dots,(n(n+1)/2)$  and  $L=1,2,\dots$  and choose that pair of (L, UCL) for which the  $ARLs(0)$  is close to the desired ARL, say  $ARL_0$ .

Table 1 is useful in choosing control parameters for a two-sided synthetic control chart for n=8.

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We note from equation (7) and from Table 1 that

- i. For a fixed value of UCL,  $ARL_s(0)$  is a decreasing function of  $L$ , and
- ii. For a fixed value of  $L$ ,  $ARL_s(0)$  is a non-decreasing function of UCL.
- iii. Due to the discrete nature of the charting statistic  $\psi_t$ , for a fixed value of  $L$ , when we get the same value of  $ARL_s(0)$  for two successive values of UCL, we choose the minimum value of UCL in order to get early out-of-control signal.

We present a side-sensitive synthetic control chart using signed-rank statistic in the following sub-section. The side-sensitivity takes into account the side of the central line where the charting statistic falls, when it falls out-side control limits. This approach has been proved to be useful in enhancing the power of the control chart.

### The Side-Sensitive Synthetic Control Chart

(Davis and Woodall, 2002) studied an important aspect of side-sensitivity, when the underlying process is normal and the mean of the process is to be monitored. They have shown that the synthetic control chart with side-sensitivity performs better than the two-sided synthetic control chart. Khilare and Shirke (2012) developed a nonparametric side-sensitive synthetic control chart using the sign statistic. Here, we study a nonparametric side-sensitive synthetic control chart based on the signed-rank statistic for the shift in the process median. The proposed chart signals an out-of-control status if two of  $(L+1)$  successive signed-rank statistic fall beyond the control limits of the chart and on only one of the sides of the center line. Here, we discuss the representation of a nonparametric side-sensitive synthetic control chart based on the signed-rank statistic for monitoring a process median by using the Markov chain approach. To write a transition probability matrix (t.p.m.) related to an absorbing state Markov chain representation of the proposed chart; first, we define required notations as follows.

We denote any signed-rank statistic  $\psi_t$  falling within control limits by "o". Any signed-rank statistic  $\psi_t$  falling above the UCL is denoted by "+", while any signed-rank statistic  $\psi_t$  falling below the LCL is denoted by "-". The symbol " $\pm$ " is introduced because of the assumption that the observation at time zero is out-side the control limits on both sides of the central line.

Let  $pc = \Pr(\text{signed-rank statistic } \psi_t \text{ is within control limits})$ ,  
 $pu = \Pr(\text{signed-rank statistic } \psi_t \text{ is on or above UCL})$  and  
 $pl = \Pr(\text{signed-rank statistic } \psi_t \text{ is on or below LCL})$ .

For  $L=2$ , the t. p. m. corresponding to a nonparametric side-sensitive control chart based on the signed-rank statistic is given below.

Table 2. Transition probability matrix for  $L=2$ .

	00	0+	0-	+0	-0	+-	-+	0±	±0	signal
00	$pc$	$pu$	$pl$	0	0	0	0	0	0	0
0+	0	0	0	$pc$	0	$pl$	0	0	0	$pu$
0-	0	0	0	0	$pc$	0	$pu$	0	0	$pl$
+0	$pc$	0	$pl$	0	0	0	0	0	0	$pu$
-0	$pc$	$pu$	0	0	0	0	0	0	0	$pl$
+-	0	0	0	0	$pc$	0	0	0	0	$pu + pl$
-+	0	0	0	$pc$	0	0	0	0	0	$pu + pl$
0±	0	0	0	0	0	0	0	0	$pc$	$pu + pl$
±0	$pc$	0	0	0	0	0	0	0	0	$pu + pl$
signal	0	0	0	0	0	0	0	0	0	1

In this case, the initial state is  $o\pm$ . For initial state, ARL is given by

$$ARL = s'(I - R)^{-1}1, \tag{8}$$

Where  $I$  is an identity matrix of order  $(L+1)^2$ ,  $1$  is a vector of ones of order  $(L+1)^2$  by one,  $R$  is a square matrix of order  $(L+1)^2$  of probabilities obtained by eliminating last row and last column of above t. p. m. and  $s' = [0, 0, 0, \dots, 0, 1, 0]$  which is a row vector of an order one by  $(L+1)^2$  with all elements zero except the element corresponding to the initial state which is unity. In the next subsection, performance study of side-sensitive synthetic control chart is given. Since the distribution of charting statistic, when the process is out-of-control is difficult to obtain, computation of the probabilities involved in the above t.p.m. is also difficult. We therefore, estimate these probabilities using simulation experiments for the selected values of  $n$  and control parameters. Computer programs using open source software R is developed for the same.

### Results and discussion

The exact distribution of the charting statistic is difficult to obtain, when there is a shift in the process median. Therefore, we use simulation to obtain the ARL values for various shifts in the process median. A simulation study based on 10000 runs is performed with  $n=8$  when the corresponding desired value is 128. The simulation study is carried for three continuous symmetric distributions namely the normal, double exponential and Cauchy. The scale parameter of the double exponential distribution is set to be  $\lambda = 1/\sqrt{2}$  to achieve a standard deviation is equal to 1. For the Cauchy distribution,  $\lambda = 0.2605$  is chosen to achieve a tail probability of 0.05 above  $\theta + 1.645$ . These three distributions are continuous symmetric about their median but have different tail behavior.

In this section, we study the performance of two-sided signed-rank, two-sided synthetic and side-sensitive synthetic control charts. The ARL results are shown in Tables 2 to 4 for the subgroup of size  $n=8$  under normal, Cauchy and double exponential distributions. For each chart  $ARL(0)=128$ .

Table 3. ARL values of two-sided synthetic and side-sensitive synthetic control charts for normal distribution when  $n=8$ .

$(\theta - \theta_0)$	Two-Sided Chart ARL(adj.) $c=36$	Two-Sided Synthetic Chart ARL(adj.) $c1=28, L=2$	Side-Sensitive Synthetic Chart ARL(adj.) $c1=28, L=2$
0.0	128.00	128.00	128.00
0.2	73.26	54.83	35.91
0.4	30.22	13.57	7.44
0.6	12.96	4.31	2.30
0.8	6.75	1.95	1.05
1.0	4.03	1.21	0.65
1.2	2.68	0.93	0.50
1.4	1.96	0.81	0.44
1.6	1.57	0.77	0.42
1.8	1.34	0.75	0.41
2.0	1.20	0.75	0.40

Table 4. ARL values of two-sided synthetic and side-sensitive synthetic control charts for Cauchy distribution when  $n=8$ .

$(\theta - \theta_0)$	Two-Sided Chart ARL(adj.) $c=36$	Two-Sided Synthetic Chart ARL(adj.) $c1=28, L=2$	Side-Sensitive Synthetic Chart ARL(adj.) $c1=28, L=2$
0.0	128.00	128.00	128.00
0.2	16.00	10.32	5.53
0.4	5.05	2.77	1.46
0.6	3.06	1.68	0.89
0.8	2.32	1.32	0.69
1.0	1.97	1.16	0.61
1.2	1.76	1.06	0.56
1.4	1.63	1.00	0.52
1.6	1.53	0.96	0.50
1.8	1.46	0.93	0.48
2.0	1.40	0.90	0.47

From Tables 3, 4 and 5, we observe that:

- The side-sensitive synthetic control chart is preferable to synthetic control chart, since the out-of-control ARL values of side-sensitive synthetic control chart are significantly better than the synthetic control chart for all distributions under study.

Table 5. ARL values of two-sided synthetic and side-sensitive synthetic control charts for double exponential distribution when  $n=8$ .

$(\theta - \theta_0)$	Two-Sided Chart ARL(adj.) $c=36$	Two-Sided Synthetic Chart ARL(adj.) $c1=28, L=2$	Side-Sensitive Synthetic Chart ARL(adj.) $c1=28, L=2$
0.00	128.00	128.00	128.00
0.20	41.19	33.91	19.15
0.40	14.60	7.23	3.99
0.60	6.85	2.69	1.48
0.80	4.08	1.54	0.84
1.00	2.81	1.10	0.60
1.20	2.16	0.92	0.50
1.40	1.77	0.84	0.46
1.60	1.54	0.80	0.43
1.80	1.37	0.77	0.42
2.00	1.27	0.76	0.41

- Performance of side-sensitive synthetic control chart is better under Cauchy distribution as compared to normal and double exponential distributions.
- Also the two-sided synthetic control chart performs significantly better than the two-sided nonparametric control chart based on signed-rank statistic for all shifts in median.

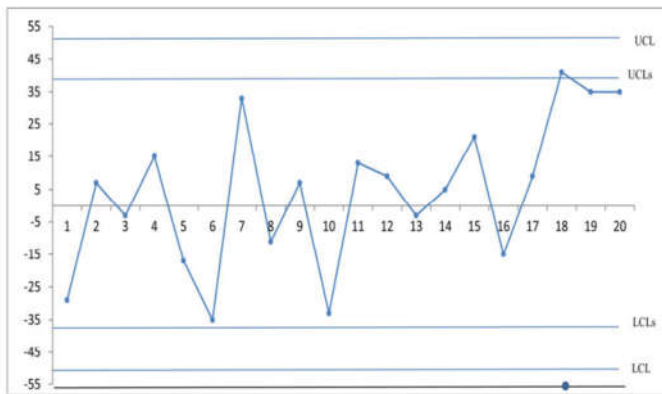
**Example**

To illustrate the construction of the two-sided synthetic signed-rank chart we use the simulated data. Fifteen samples of size ten each were randomly generated from normal distribution with mean zero and standard deviation 1 and then, five samples of size ten each were randomly generated from normal distribution with mean 0.4 and standard deviation 1. Upper and lower control limits of two-sided signed-rank chart are  $UCL=52$  and  $LCL=-52$  respectively while upper and lower control limits of two-sided synthetic signed-rank chart are  $UCLs=38$  and  $LCLs=-38$  respectively. For synthetic chart the lower control limit for the CRL chart is  $L=2$ . Table 6 contains the values of the signed-rank statistic based on the simulated data.

Table 6. Values of signed-rank statistic for 20 samples of size 10 each.

Sample No.	$\psi_t$	Sample No.	$\psi_t$
1	-29	11	13
2	7	12	9
3	-3	13	-3
4	15	14	5
5	-17	15	21
6	-35	16	-15
7	33	17	9
8	-11	18	41
9	7	19	35
10	-33	20	35

Fig. 1. Two-sided signed-rank and two-sided synthetic signed-rank charts.



From Figure 1, it is observed that no charting statistic falls on or outside the control limits of the two-sided signed-rank chart and hence it does not give an out-of-control signal. Though a charting statistic corresponding to sample no. 18 falls above the upper control limit, the corresponding CRL value is 18 which is greater than  $L=2$  and hence the synthetic two-sided signed-rank chart also does not give an out of control signal.

## Conclusion

In this article, two nonparametric control charts are proposed namely, two-sided synthetic and side-sensitive synthetic control charts based on signed-rank statistic. Both the proposed control charts performs superior than the signed-rank control chart. ARL values of synthetic and side-sensitive synthetic control charts are respectively at least 41% and 69% less as compared to the signed-rank control chart to detect small to moderate shifts. Also, ARL values of the side-sensitive synthetic control chart are at least 44% less than the synthetic control chart. The study revealed that the side-sensitive synthetic control chart has better power to detect early out-of-control signal. The operations and design procedure of side-sensitive synthetic control chart is similar to the synthetic control chart. Both the proposed control charts are simple and easy for practitioners in nonparametric setup.

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